The randomized level-set method to model turbulence effects in wildland fire propagation

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Abstract

The wildland fire propagates at the ground level and then strongly depends on the dynamics of the Atmospheric Boundary Layer, whose flow is turbulent in nature. Turbulence is amplified by the forcing due to the fire-atmosphere coupling and by the appearing of the fire-induced flow. Turbulence randomly transports the hot air mass that can pre-heat and then ignite the area around and ahead the fire. The fire front position gets therefore a random character. The level-set method for tracking fire line contour is randomized according to the probability density function of the turbulent displacement of the hot air particles. The proposed model emerges to be suitable to simulate a moving fire front displaced by the pre-heating action of the hot air mass itself diffused by the turbulence. This mechanism allows the simulation of the fire overcoming a firebreak zone, a situation that the level-set method can not resolve.

Keywords: fire front propagation, level-set method, randomized level-set method, fire-atmosphere coupling, fire-induced flow, turbulence

1. INTRODUCTION

The wildland fire propagation is a multi-scale, as well as a multi-physics, process strongly influenced by the atmospheric wind. Since the fire front propagates at the ground level it is driven by the dynamics of the Atmospheric Boundary Layer (ABL), whose large scale motion influences the fire front velocity while the small scale motion is related to turbulent flows that play a fundamental role in wildfire spread as demonstrated in [1]. Actually, when a wildland fire occurs, the ABL emerges to be forced also by the fire-atmosphere coupling and turbulence increases. Close to the fire front, turbulence is increased also by the fire-induced flow. To model the effects of turbulence on the fire front propagation, the suitability of a recent approach based on the statistical distribution of the level-set contour [2] is here investigated.

The level-set is a method to track moving interfaces [3] that has been recently applied to study the fire front contour in wildland fire propagation [4], [5], [6]. It allows the representation of the burning region on a simple cartesian grid and the flexible implementation of various ignition modes. Moreover, this method is particularly appropriate to handle problems that arise from propagation of wildfires since leads to an accurate calculation of the front normal vector, which is necessary to compute the burning velocity. The level-set method can automatically deal with topological changes that can occur during the fire spreading, as the merging of separate flame fronts or the formation of unburned “islands”.

The level-set method for tracking fire line contour is randomized by considering a distribution of the contour according to the probability density function (PDF) of the turbulent displacement of the hot air particles. The accounting for the effect of turbulence on the fire propagation improves the usefulness of the operational models and thereby increases the firefighting safety and in general the efficiency of the efforts for the fire suppression and control.

The paper is organized as follows. In paragraph 2, the random level-set method is introduced and the proposed model described. In paragraph 3, the model is applied to predict the fire front motion when the pre-heating induced by the hot turbulent flow is taken into account and also to successfully tackle the realistic situation when the ordinary level-set method fails: fire overcoming of firebreaks. Finally in paragraph 4 some conclusions on the performance of the model are given.

2. MODEL DESCRIPTION

The concept of the *randomized level-set method* is founded on the idea that, due to the turbulence, the fire front contour cannot be assumed to be deterministic, so that the resulting *effective* fire front propagates according to the statistical distribution of a random trajectory. Hence, a statistically distributed fire line follows.

Let $\bar{x}(t,\bar{x}_0)$be a deterministic trajectory with initial condition $\bar{x}_0$, i.e., $\bar{x}(0,\bar{x}_0)=\bar{x}_0$, and driven solely by the rate of spread $V(\bar{x},t)$. Moreover, let $\phi(\bar{x},t)$be the function with values 1and 0such that$\phi(\bar{x},t)=1$ marks the burned area $\Omega(t)$, i.e., $\Omega(t)=\{\bar{x},t\colon\phi(\bar{x},t)=1\}$, and $\phi(\bar{x},t)=0$marks the unburned area, i.e., $x\notin\Omega(t)$. Let $X^\omega(t,\bar{x}_0)=\bar{x}(t,\bar{x}_0)+\sigma^\omega$be the ω-realization of a random trajectory driven by the noise σ, with average value $\langle X^\omega(t,\bar{x}_0)\rangle=\bar{x}(t,\bar{x}_0)$and the same fixed initial condition $X^\omega(0,\bar{x}_0)=\bar{x}_0$in all realizations. Hence, the ω-realization of the fire line contour follows to be

$$\phi^\omega(x,t)=\int\phi(\bar{x}_0,0)\delta(x-X^\omega(t,\bar{x}_0))d\bar{x}_0. \tag{1}$$

Since the trajectory $\bar{x}(t,\bar{x}_0)$is time-reversible, i.e., the Jacobian of its transformation $\bar{x}(t,\bar{x}_0)=F_t(\bar{x}_0)$is $\frac{d\bar{x}_0}{d\bar{x}}=1$, then Formula (1) becomes

$$\phi^\omega(x,t)=\int\phi(\bar{x},t)\delta(x-X^\omega(t,\bar{x}))d\bar{x}. \tag{2}$$

Finally, after averaging, the effective fire front contour emerges to be determined as

$$\begin{aligned}\langle\phi^\omega(x,t)\rangle=\langle\int\phi(\bar{x},t)\delta(x-X^\omega(t,\bar{x}))d\bar{x}\rangle&=\int\phi(\bar{x},t)\langle\delta(x-\bar{X}^\omega(t,\bar{x}))\rangle d\bar{x}\\&=\int\phi(\bar{x},t)p(x;t\mid\bar{x})d\bar{x}\\&=\int_{\Omega(t)}p(x;t\mid\bar{x})d\bar{x}=\phi_{eff}(x,t),\end{aligned} \tag{3}$$

where $p(x;t\mid\bar{x})=p(x-\bar{x};t)$is the PDF of the turbulent dispersion of the hot flow particles with average position $\bar{x}$.

Formula (1) has been originally proposed to model the progress variable in turbulent premixed combustion [2]. The evolution equation of the effective fire front $\phi_{eff}(x,t)$is [2]

$$\frac{\partial\phi_{eff}}{\partial t}=\int_{\Omega(t)}\frac{\partial p}{\partial t}d\bar{x}+\int_{\Omega(t)}\nabla_{\bar{x}}\cdot[V(\bar{x},t)p(x-\bar{x};t)]d\bar{x}, \tag{4}$$

that for a deterministic motion, i.e., $p(x-\bar{x};t)=\delta(x-\bar{x})$, reduces to the ordinary level-set front propagation [2]. This fire line distribution permits to model the pre-heating of the area ahead the flame front. Points $x$such that $\phi_{eff}(x,t)>0.5$are marked as burnt.

The model is completed by a law for the ignition due to the pre-heating induced by the hot turbulent flow. Let $T(x,t)$be the temperature field, then the superposition of hot air in an unburned point x, i.e., $\phi_{eff}(x,t)\leq 0.5$, for a temporal interval Δtmakes the temperature rise up to the ignition value $T(x,\Delta t)=T_{ign}$.

Assuming that the temperature grows according to

$$\frac{\partial T(x,t)}{\partial t}=\phi_{eff}(x,t)\frac{T_{ign}-T(x,0)}{\tau},T\leq T_{ign}, \tag{5}$$

then, for a given characteristic ignition delay τ, it holds

$$\tau=\int_0^{\Delta t}\phi_{eff}(x,\xi)d\xi. \tag{6}$$

3. NUMERICAL SIMULATIONS

3.1. Numerical simulation set-up

Simulations are performed assuming an isotropic bi-variate Gaussian PDF

$$p(x-\bar{x};t)=\frac{1}{2\pi\sigma^2}\exp\left\{-\frac{(x-\bar{x})^2+(y-\bar{y})^2}{2\sigma^2}\right\} \tag{7}$$

where $\sigma^2$is the particle displacement variance related to the turbulent diffusion coefficient D, i.e.,$\langle(x-x)^2\rangle=\langle(y-\bar{y})^2\rangle=\sigma^2=2Dt$.

The rate of spread $V(\bar{x},t)=V(\bar{x},t)\hat{n}$, where $\hat{n}=-\nabla\phi/||\nabla\phi||$is the outcoming normal, is calculated from the well known Rothermel semi-empirical formula [7]

$$V(\bar{x},t)=V_0(1+f_W+f_S) \tag{8}$$

where $V_0$is the spread rate in the absence of wind, $f_W$is the wind factor and $f_S$is the slope factor. The reader is referred to fireLib and Fire Behaviour SDK documentation (http://fire.org/) and to [6] for a full description of V_0, $f_W$and f_S.

However, to best highlight the model performance, the simulations are performed in the most simple case with no wind, no slope and short grass fuel, i.e., NFFL (Northern Forest Fire Laboratory) Model 1, and with a unique dead fuel moisture of the type 1-hour dead fuel moisture (i.e., those fuels whose moisture content reaches equilibrium with the surrounding atmosphere within one hour) that is stated equal to 0.1[lb water / lb fuel]. Lenght and elapsed time are expressed in feets [ft] and minutes [min], respectively.

3.2. Pre-heating effects on the fire-front propagation

Two competing ignition mechanisms for fire front propagation are considered: the flame reaching a certain place, or the hot air mass heating an area so much that it burns after an ignition delay Δt. Then, if the ignition delay $\Delta t$is short enough, places heated by the hot air can burn before than the fire flame is arrived and the effective fire front velocity results to be increased. The stronger is the turbulence, the more distant from the fire flame the hot air is diffused.

Figures 1 and 2 show that strong turbulence and short ignition time delay generate a faster fire front propagation whereas for long ignition delay the effects of the pre-heating are negligible. In the plots, the strong or weak effect of pre-heating is embodied by the time that the fire needs to reach the boundaries of the considered domain.

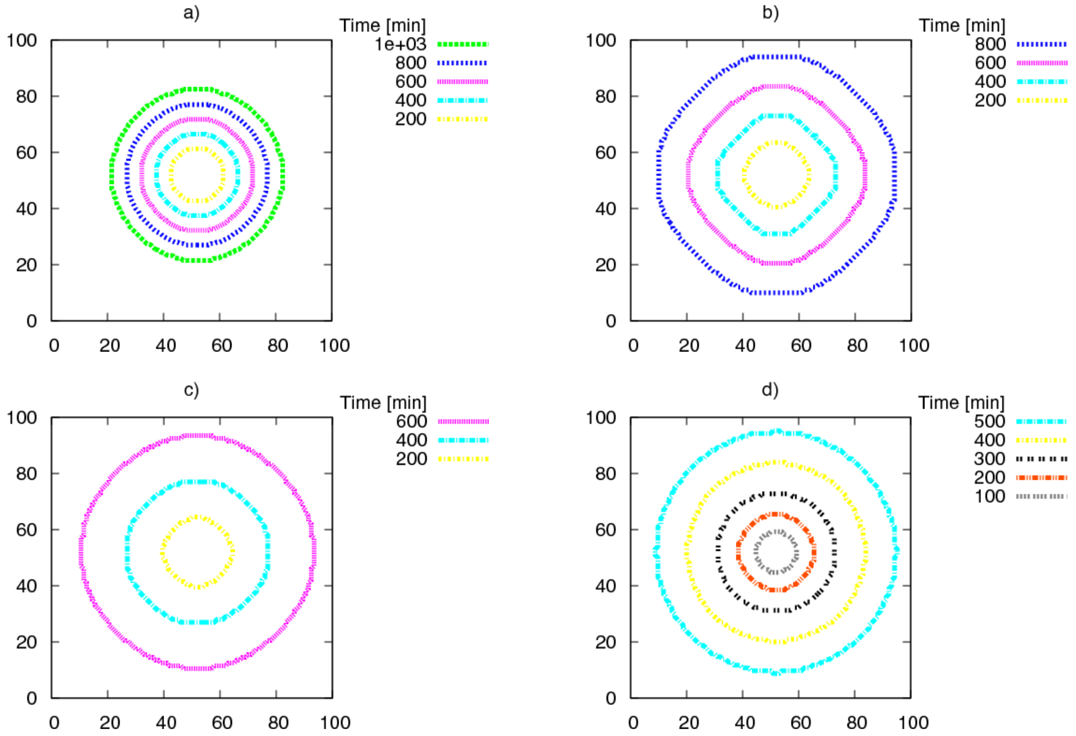

Figure 1. Evolution in time of the fire line contour, when $\tau = 10[\mathrm{min}]$, *for the level-set method a) and for the random level-set method with increasing turbulence: b)* $D = 25[\mathrm{ft}]^2[\mathrm{min}]^{-1}$, *c)* $D = 100[\mathrm{ft}]^2[\mathrm{min}]^{-1}$, *d)* $D = 225[\mathrm{ft}]^2[\mathrm{min}]^{-1}$. *The domain axes are expressed in feets and the colors refer to the elapsed time in minutes according to the column on the right*

3.3. Fire front overcoming firebreaks

The realistic situation in which the level-set method fails is the simulation of a fire that overcomes a firebreak. In fact, the firebreak is a zone without fuel and is modeled as an area in which the rate of spread of the fire is $V(x, t) = 0$, causing the fire to stop. However, the hot air mass can overcome the firebreak and ignite an area over it, so that a new fire front starts. In Figure 3 it is shown the suitability of the proposed model to simulate the hot air that overcomes a firebreak so that the fire front passes on. The stronger is the turbulence, the earlier is the ignition behind the firebreak. The spotting phenomenon can be modeled with a similar technique.

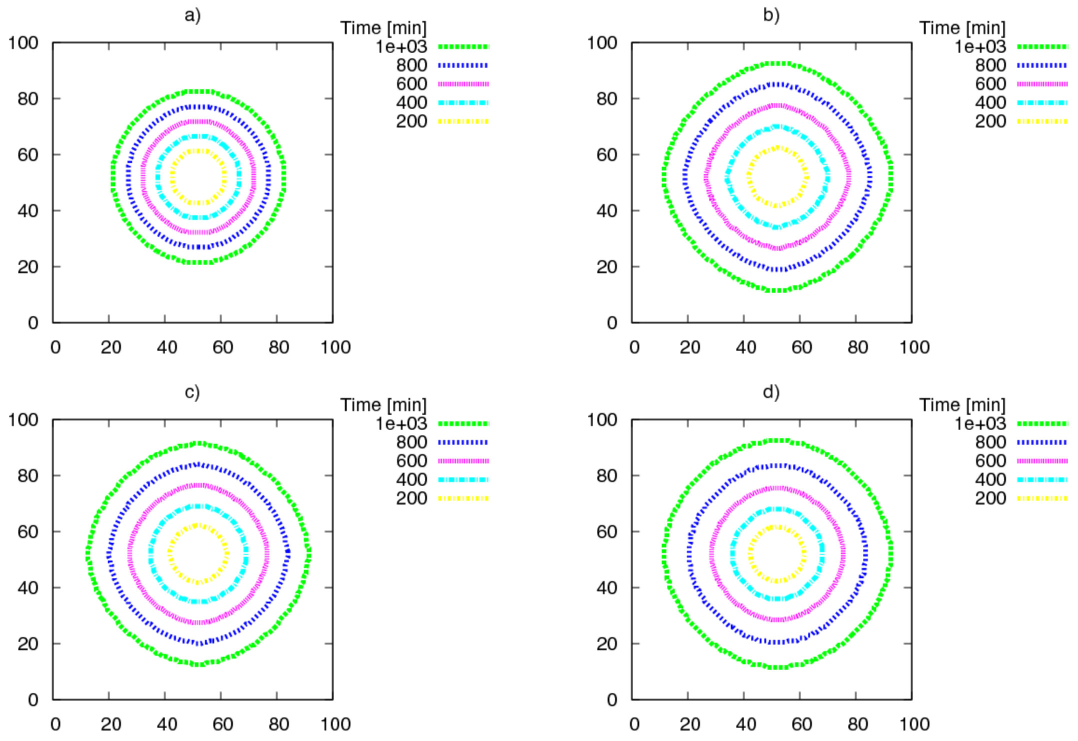

Figure 2. The same as in Figure 1 but when $\tau = 50[\mathrm{min}]$.

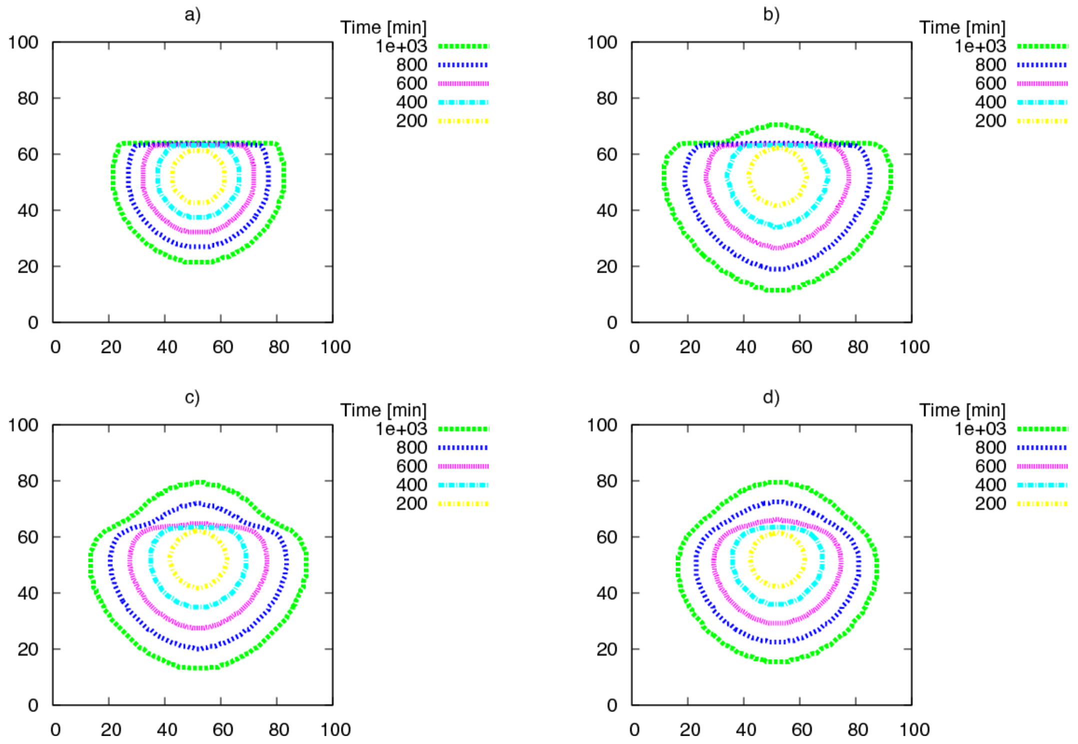

Figure 3. The same as in Figure 1 but in the presence of a firebreak and when $\tau = 100[\mathrm{min}]$.

4. CONCLUSION

A new formulation for modeling the wildland fire front propagation is proposed. It includes small scale processes driven by the turbulence generated by the ABL dynamics, which is increased by the fire-atmosphere coupling, and by the fire-induced flow. It is based on the randomization of the level-set method for tracking fire line contour by considering a distribution of the contour according to the PDF of the turbulent displacement of hot air particles.

This formulation is emerged to be suitable, more than the ordinary level-set approach, to model the following two dangerous situations: i) the faster propagation of the fire line as a consequence of the pre-heating action by the hot air mass and ii) the overcoming of a break-fire by the fire because of the diffusion of the hot air behind it.

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