Recent History of Fractional Calculus

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The fractional calculus started from some speculations of G.W. Leibniz (1695, 1697) and L. Euler (1730), and it has been developed progressively up to now. A list of mathematicians, who have provided important contributions up to the middle of the twentieth century, includes P.S. Laplace (1812), S. F. Lacroix (1819), J. B. J. Fourier (1822), N. H. Abel (1823–1826), J. Liouville (1832–1873), B. Riemann (1847), H. Holmgren (1865–1867), A. K. Grunwald (1867–1872), A. V. Letnikov (1868–1872), H. Laurent (1884), P. A. Nekrassov (1888), A. Krug (1890), J. Hadamard (1892), O. Heaviside (1892–1912), S. Pincherle (1902), G. H. Hardy and J. E. Littlewood (1917-1928), H. Weyl (1917), P. Lévy (1923), A. Marchaud (1927), H. T. Davis (1924-1936), E. L. Post (1930), A. Zygmund (1935-1945), E. R. Love (1938-1996), A. Erdelyi (1939-1965), H. Kober (1940), D. V. Widder (1941), M. Riesz (1949), W. Feller (1952).

Only since the Seventies has fractional calculus been the object of specialized conferences and treatises. For the first conference the merit is due to B. Ross who, shortly after his Ph.D. dissertation on fractional calculus, organized the First Conference on Fractional Calculus and its Applications at the University of New Haven in June 1974, and edited the proceedings. For the first monograph the merit is ascribed to K. B. Oldham and I. Spanier who, after a joint collaboration begun in 1968, published a book devoted to fractional calculus in 1974.

In recent years considerable interest in fractional calculus has been stimulated by the applications it finds in different areas of applied sciences like physics and engineering, possibly including fractal phenomena. Now there are more books of proceedings and special issues of journals published that refer to the applications of fractional calculus in several scientific areas including special functions, control theory, chemical physics, stochastic processes, anomalous diffusion, rheology. Several special issues appeared in the last decade which contain selected and improved papers presented at conferences and advanced schools, concerning various applications of fractional calculus. Already since several years, there exist two international journals devoted almost exclusively to the subject of fractional calculus: Journal of Fractional Calculus (Editor-in-Chief: K. Nishimoto, Japan) started in 1992, and Fractional Calculus and Applied Analysis (Managing Editor: V. Kiryakova, Bulgaria) started in 1998. Recently the new journal Fractional Dynamic Systems has been announced to start in 2010.

The authors believe that the volume of research in the area of fractional calculus will continue to grow in the forthcoming years and that it will constitute an important tool in the scientific progress of mankind.

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