HYPERBOLIC CONSERVATION LAWS AND APPLICATIONS TO THE DAM BREAK PROBLEM

Massimo Tomirotti

Department of Civil Engineering – University of Brescia
Via Branze, 38 – 25123 BRESCIA (Italy)

Floods resulting from the sudden collapse of a dam (*Dam-Break*) are often characterized by the formation of shock waves due to irregular bed topography and nonzero tailwater depth. The mathematical description of these phenomena is usually accomplished by means of 1D St.Venant equations written in conservation form:

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}=\mathbf{S},\ \ \mathbf{U}=\begin{pmatrix}A\\Q\end{pmatrix},\ \ \mathbf{F}=\begin{pmatrix}Q\\ \dfrac{Q^2}{A}+gI_1\end{pmatrix},\ \ \mathbf{S}=\begin{pmatrix}0\\ gA\left(S_0-S_f\right)+gI_2\end{pmatrix},\ \ S_f=\frac{n^2Q|Q|}{A^2R^{4/3}}, \tag{1}$$

with Q=discharge, A=wetted area, R=hydraulic radius, x=distance along the channel, t=time, g=gravitational acceleration, S_0=bottom slope, S_f=friction slope calculated according to Manning equation and

$$I_1=\int_0^h (h-\eta)\sigma(x,\eta)d\eta, \qquad I_2=\int_0^h (h-\eta)\frac{\partial \sigma}{\partial x}d\eta, \tag{2}$$

where h = water depth and $\sigma(x,\ \eta)$ = width of the cross-section at height η above the bottom.

In the last decade many works have been carried out in the field of numerical solution of 1D and 2D St. Venant Equations, mainly devoted to the treatment of source-terms and to accurately capture discontinuities (Molinaro & Natale, 1994). Verification of the capabilities of the numerical schemes are often performed comparing the computed results with analytical solutions; only few experiments in literature concern formation and propagation of shock waves (Chervet & Dalléves, 1970; Bellos et al., 1992).

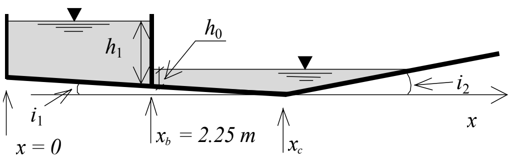

Fig. 1. Channel geometry.

Test	n	i_1 (%)	i_2 (%)	x_c (m)	h_1 (m)	h_0 (m)	Test	n	i_1 (%)	i_2 (%)	x_c (m)	h_1 (m)	h_0 (m)
1	0.01	+1	–9	3.40	0.210	0	4	0.025	0	–10	3.50	0.292	0
2	0.01	0	–10	3.40	0.250	0	5	0.025	+2	–8	3.50	0.250	0
3	0.01	0	–10	3.40	0.250	0.045	6	0.025	0	–10	3.50	0.292	0.050

Table I. Test conditions.

In the following experimental results of *1D Dam-Break* flows are compared with those obtained from a mathematical model based on the MacCormack *shock-capturing* scheme (see Aureli et al., 1999, 2000 for details). Two versions of the model have been implemented, in which artificial dissipation terms are computed according both to Jameson formulation (Jameson, 1981) and *TVD* approach adopting Van Leer *limiter* function (Harten, 1983).

Tests were carried out at Department of Civil Engineering of Parma University in a tilting laboratory flume rectangular in section, 1.0 m wide, 0.5 m high and 7.0 m long. Experimental tests were designed to induce shock formation and propagation and wetting and drying conditions (Fig. 1 and Table I).

Measurements of water depth were made at four sections along the flume, including the dam site, based on video recordings of the flow. Figs. 2-3 show numerical and experimental stage hydrographs for test cases N.2 and N.4, that could be considered representative of the whole set.

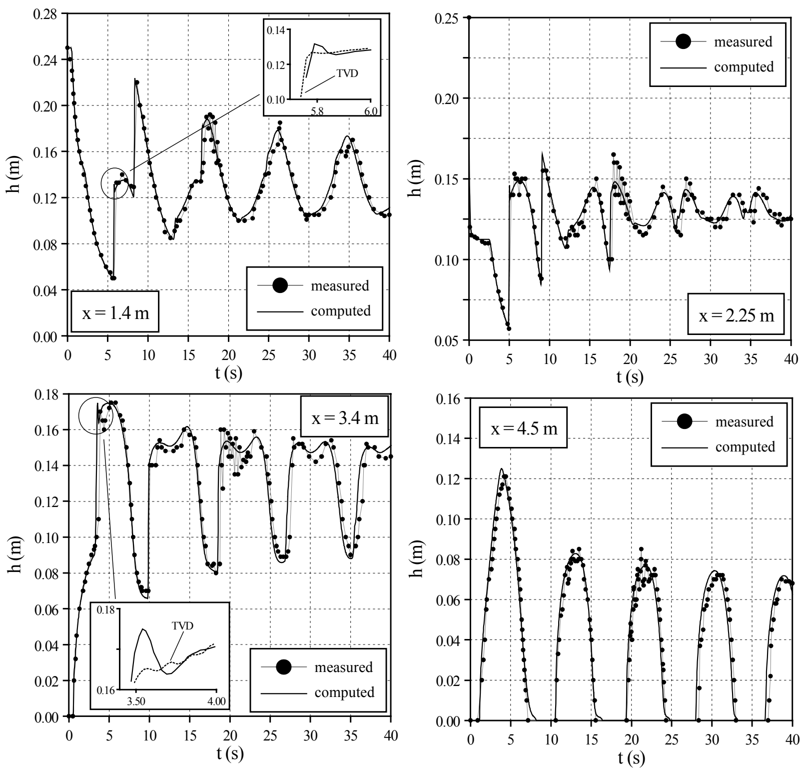

Fig. 2. Computed and measured stage hydrographs for test case 2.

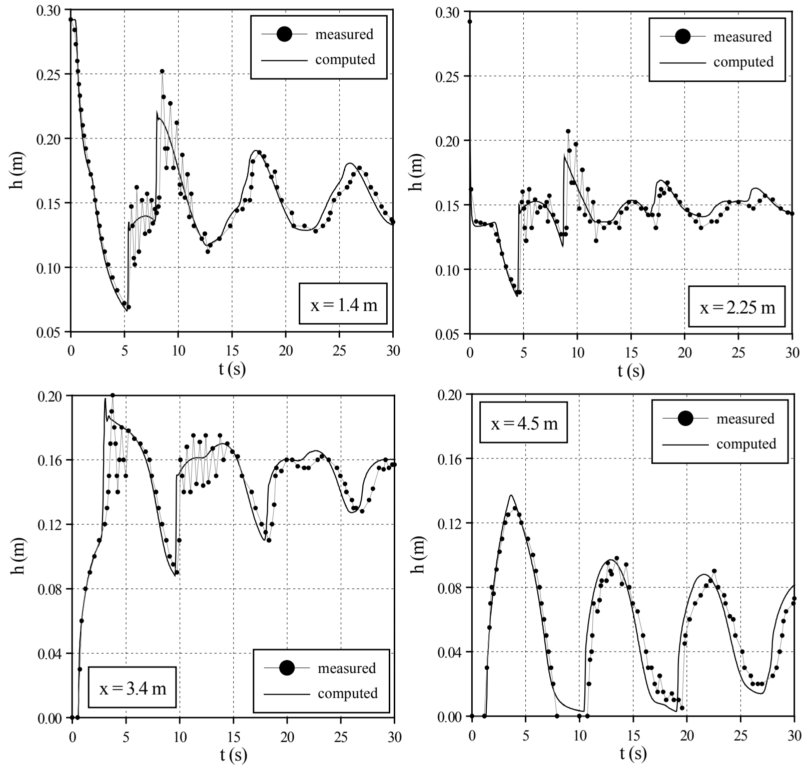

Fig. 3. Computed and measured stage hydrographs for test case 4.

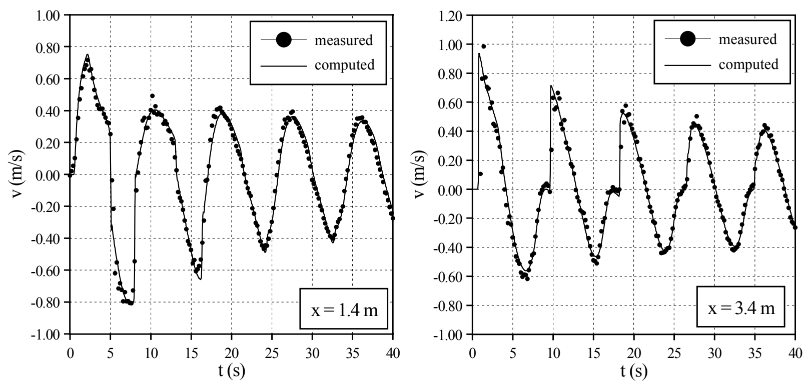

Fig. 4. Computed and measured velocities for test case 3.

After the opening of the gate, the velocity of the wetting front progressively decreases and a shock originates slightly downstream of the beginning of the adverse slope. The shock wave moves upstream, reflects on the wall and starts to propagate downstream, vanishing nearly in the same section where it was initially formed. In the meantime on the adverse slope a wetting-drying front passes. The whole sequence is repeated but, after the reflection on the upstream wall, the shock is followed by a surface wave train. In the rough test case the behaviour is similar but surface waves already follow the first shock reflection.

Solid lines refer to numerical solution obtained by Jameson formulation of artificial dissipation; *TVD* version of the model provides almost identical results except around discontinuities, that are more sharply captured.

Comparison of experimental and numerical results shows a very good agreement as a whole. Shocks, reverse flows and wetting and drying fronts are well predicted. Of course, surface waves with non zero vertical velocity components cannot be handled by the model based on 1D St. Venant equations. Anyway, the numerical solution gives a satisfactory representation of the average depth.

Velocity measurements were also accomplished by means of an Acoustic Doppler Velocity meter (ADV Nortek). The control volume was placed on the symmetry axis near the bottom of the flume. Fig. 4 shows calculated and experimental velocities for test case N.3 at sections x=1.40 *m* (inside the reservoir) and x=3.40 *m* (at the beginning of the adverse slope). Numerical results are very close to experiments, even if the first refer to *mean velocity* and the second to *point velocity* near the bottom. This also suggests that velocity distribution across the section is almost uniform, as proved by the observation of the suspended moving particles. In conclusion, the agreement between experimental and computed results confirms the validity of the numerical model, even in situations in which St. Venant hypothesis are not completely verified.

References

Aureli F., Mignosa P. & Tomirotti M.: Dam-Break flows in presence of abrupt bottom variations, Proceedings of The XXVIII IAHR International Congress, Graz August 22-27 1999, Graz, Austria, 1999.

Aureli F., Mignosa P. & Tomirotti M.: Numerical Simulation and experimental verification of Dam-Break flows with shocks, *Journal of the Hydraulic Research*, **38**, 3, 2000, pp. 197-205.

Bellos V., Soulis J.V. & Sakkas J.G.: Experimental investigation of two-dimensional dam-break induced flows, *Journal of the Hydraulic Research*, **30**, 1, 1992, pp. 47-63.

Chervet A. & Dallèves P.: Calcul de l'onde de submersion consecutive a la rupture d'un barrage, *Schweizerische Bauzeitung*, **88**, 1970, pp. 420-432.

Harten A.: High Resolution Schemes for Hyperbolic Conservation Laws, *Journal of Computational Physics*, **49**, pp. 357-393, 1983.

Jameson A.: Transonic Airfoil calculations using the Euler equations, *Numerical Methods in Aeronautical Fluid Dynamics*, ed. P.L. Roe, Academic Press, New York, 1982.

MacCormack R.W.: The effect of viscosity in hypervelocity impact cratering, *AIAA Paper* 69-354, 1969.

Molinaro, P. & Natale, L. (eds.): *Modelling of Flood Propagation over Initially Dry Areas*, ASCE, New York, 1994.