INTERDISCIPLINARY WORKSHOP
"FROM WAVES TO DIFFUSION AND BEYOND"
BOLOGNA, Italy : December 20, 2002

SELF-SIMILAR STOCHASTIC PROCESSES

Claudia MANZONI[1] and Ombretta PINAZZA[2]

[1] Mathematics Institute, University of Warwick
Coventry CV4 7AL England, UK
manzoni@maths.warwick.ac.uk

[2] INFN - Istituto Nazionale di Fisica Nucleare, Sezione di Bologna
Via Irnerio 46, I-40126 Bologna
Ombretta.Pinazza@bo.infn.it

This work is concerned with the analysis of self-similar stochastic processes, where statistical self-similarity refers to invariance in distribution under a suitable change of scale. In particular, emphasis is placed upon those processes which, being non-local in time, can display long memory and/or long-range correlation effects, and are connected to anomalous diffusion. Whereas normal diffusion is characterized by a mean-square displacement which is asymptotically linear in time according to Einstein law, in anomalous diffusion, a phenomenon met in a number of physical, chemical and biological systems, we find appreciable deviations from this law.
Fractional Brownian motion (fBm) and time fractional diffusion are both self-similar stochastic processes, that display deviations from Einstein law. The first one is Gaussian, non-stationary, with stationary increments; the second is non-Gaussian, non-stationary with increments that are also non-stationary. The class of self-similar stochastic processes is related, via a simple one-to-one connection, called Lamperti transformation, to another important class of stochastic processes, the stationary processes (statistically invariant under time shifts). The Lamperti transformation, which essentially consists of a proper warping of the time axis, is a bijective invertible map which connects any self-similar process with a stationary counterpart. Although introduced by Lamperti in 1962, this fundamental result has further received little attention until a recent past when it has been rediscovered and appreciated as an important tool allowing researchers to apply the extensive body of knowledge about stationary processes to examine the structure of the associated self-similar processes.

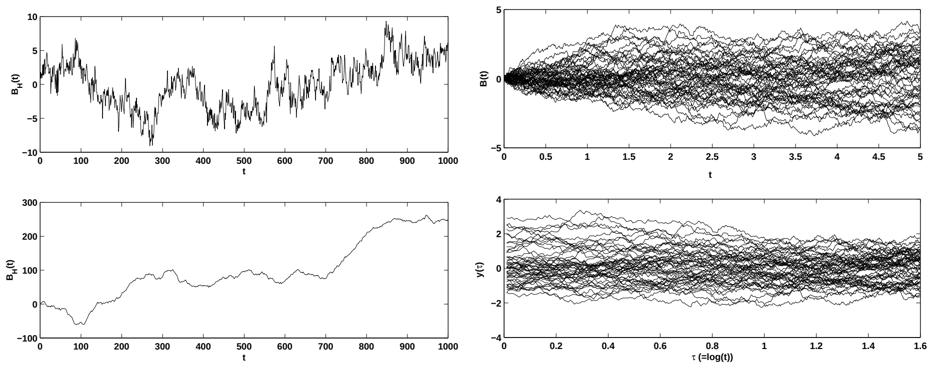

Figure 1: (Left) Two paths of fBm with $H = 0.25$ (top) and $H = 0.75$ (bottom); (Right) Lampertization of Brownian motion

References

[1] P. Flandrin and P.O. Amblard, From stationarity to self-similarity and back: Variations on the Lamperti transformations, in G. Rangarajan and M. Ding (Editors), *Long Range Dependent Processes: Theory and Applications*, Springer Verlag, to appear (2003).

[2] R. Gorenflo, F. Mainardi, D. Moretti, G. Pagnini and P. Paradisi, Discrete random walk models for space-time fractional diffusion, *Chemical Physics* **284** (2002) 521-544.

[3] J.W. Lamperti, Semi-stable stochastic processes, *Trans. Amer. Math. Soc.* **104** (1962) 62-78.

[4] B.B. Mandelbrot and J.W. Van Ness, Fractional Brownian motions, fractional noises and applications, *SIAM Review* **10** (1968) 422-433.

[5] C. Manzoni, *Analysis of Self-Similar Stochastic Processes*, Thesis for Degree in Physics, University of Bologna, October 2002, in Italian. [Supervisors: Prof. F. Mainardi and Dr. O. Pinazza]

[6] O. Pinazza, *Fractal Dimension and Fractional Calculus: Theory and Applications*, Thesis for Degree in Physics, University of Bologna, July 1993, in Italian. [Supervisors: Prof. F. Mainardi and Prof. S. Rambaldi]

[7] S. Rambaldi and O. Pinazza, An accurate fractional Brownian motion generator, *Physica A* **208** (1994) 21-30.