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Exact solution $u(x, t) = x^\beta e^{-\lambda x - t} / \Gamma(1 + \beta)$ vs Crank-Nicolson solution (extrapolated) to

$$\partial_t p(x, t) = a(x) \mathbb{D}_x^{\alpha, \lambda} p(x, t) + c(x, t)$$

on $0 < x < 1$ with $\alpha = 1, 6$, $\lambda = 2$, $\beta = 2.8$, $p(x, 0) = x^\beta e^{-\lambda x} / \Gamma(\beta + 1)$, $p(0, t) = 0$, $p(1, t) = e^{-\lambda - t} / \Gamma(\beta + 1)$, $a(x) = x^\alpha \Gamma(1 + \beta - \alpha) / \Gamma(\beta + 1)$, and $c(x, t) = c_1(x, t) e^{-\lambda x - t} \Gamma(1 + \beta - \alpha) / \Gamma(\beta + 1)$ where

$$c_1(x, t) = \frac{(1 - \alpha) \lambda^\alpha x^{\alpha + \beta}}{\Gamma(\beta + 1)} + \frac{\alpha \beta \lambda^{\alpha - 1} x^{\alpha + \beta - 1}}{\Gamma(\beta)} - \frac{2x^\beta}{\Gamma(1 + \beta - \alpha)}.$$

Δt	Δx	CN Error	Rate	RE Error	Rate
1/10	1/50	7.7738×10^{-5}	—	2.8514×10^{-6}	—
1/20	1/100	3.8353×10^{-5}	2.03	7.2120×10^{-7}	3.95
1/40	1/200	1.9055×10^{-5}	2.01	1.8157×10^{-7}	3.97

Question: Is this solution unique?