Unsolved or Open Problems in Fractional Calculus, June 28, 2014

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"Open Problems in Diffusion-Weighted Medical Imaging"

Overview: The medical and biomedical research communities who use diffusion-weighted magnetic resonance imaging (MRI) are trying to assess changes in tissue structure by monitoring the diffusion of tissue water using special MR pulse sequences that tag the precession of spins on the hydrogen in water. This is difficult to do because tissue in heterogeneous, complex, anisotropic and multi-scale, multi-compartmental and multi-component. Fractional calculus provides a new way of reducing the dimensionality of the problem – particularly when the diffusion is viewed as a continuous time random walk problem. For integer order systems, the Bloch-Torrey equation provides a good way to describe anisotropic diffusion and is the basis for diffusion tensor imaging (DTI). My group and colleagues (A. Hanyga, M. Meerschaert) are trying to extend the fractional calculus models of diffusion to account for spatial variation in fractional order, e.g., fractional order vector calculus or order $\alpha(x,y,z)$. In order to proceed we also need a consistent way of describing the directional dependence of α as a scalar, a vector and perhaps even as a tensor, which is the case for the diffusion coefficient in anisotropic tissues such as muscle, ligaments, and brain white matter. Below are some recent references to papers in which we have begun to approach this problem.

References:

1. Ingo, C., Magin, R. L., Colon-Perez, L., Triplett,W. & Mareci, T. H. 2014 On random walks and entropy in diffusion weighted magnetic resonance imaging studies of neural tissue. Magn. Reson. Med., 71, 617–627.
2. Hanyga, A. & Seredynska, M. 2012 Anisotropy in high-resolution diffusion-weighted MRI and anomalous diffusion. Journal of Magnetic Resonance, 220, 85–93. doi:10.1016/j.jmr.2012.05.001.
3. Hanyga, A. 2001 Multi-dimensional solutions of space-fractional diffusion equations. Proc. Roy. Soc. London A, 457, 2993–3005.
4. Baeumer, B. & Meerschaert, M. 2001 Stochastic solutions for fractional Cauchy problems. Fract. Calc. Appl. Anal., 4, 481–500.
5. Magin, R. L., Abdullah, O., Baleanu, D. & Zhou, X. J. 2008 Anomalous diffusion expressed through fractional order differential operators in the Bloch-Torrey equation. J Magn Reson, 190, 255–270.
6. Meerschaert, M. M. & Sikorskii, A. 2012 Stochastic Models for Fractional Calculus. Walter De Gruyter, Berlin.
7. GadElkarim, J. J., Magin, R. M., Meerschaert, M. M., Capuani, S., Palombo, M., Kumar, A. & Leow, A. D. 2013 Directional behavior of anomalous diffusion expressed through a multidimensional fractionalization of the Bloch-Torrey equation. IEEE J. on Emerging and Selected Topics in Circuits and Systems, 3, 432–441.