Open Problem:

To derive the eigenvalues and the eigenfunctions of the time-independent (at least one-dimensional) space-fractional Schrödinger equation of the order $\alpha,\ 1 < \alpha < 2$ in the form

$$\left((-\Delta)^{\alpha/2} + V(x)\right)\phi(x) = \lambda\phi(x),\ x \in \boldsymbol{R},\ 1 < \alpha < 2$$

with the quantum Riesz fractional derivative

$$(-\Delta)^{\alpha/2}\phi(x) := (\mathcal{F}^{-1}\left\{|\kappa|^{\alpha}(\mathcal{F}\phi)(\kappa)\right\})(x)$$

in the case of the infinite potential well

$$V(x) = \begin{cases} 0, & |x| < L, \\ \infty, & |x| \geq L. \end{cases}$$

Conjecture:

The eigenfunctions of the space-fractional Schrödinger equation in the case of the infinite potential well (or the eigenfunctions of the fractional Laplacian on a finite interval) cannot be expressed in terms of the known elementary and special functions and should be treated as new special functions.

Literature:

1. S.S. Bayin: On the consistency of the solutions of the space fractional Schrödinger equation. J. Math. Phys. 53(2012), 042105.
2. M. Jeng, S.-L.-Y. Xu, E. Hawkins, and J.M. Schwarz: On the nonlocality of the fractional Schrödinger equation. J. Math. Phys. 51(2010), 062102.
3. M. Kwasnicki, Eigenvalues of the fractional Laplace operator in the interval. Journal of Functional Analysis 262(2012), 2379-2402.
4. Yu. Luchko: Fractional Schrödinger equation for a particle moving in a potential well. J. Math. Phys. 54(2013), 012111.