Quo vadimus?

Possible directions by Richard Herrmann, 11 July, 2014.

Fractional Group theory

Group theory has been developed from its gentle beginnings (Abel, Galoise, Lie) to a powerful tool for an understanding of symmetries in complex scenarios. The fractional extension of a standard Lie-groups describes extended symmetries and therefore allows to describe more complex phenomena and symmetries, which go beyond the standard and even can bridge different standard Lie-groups.

One example is the fractional extension of SO(n), which besides rotational also describes vibrational degrees of freedom[1].

A promising field of future research is an extension to more complex groups as well as an investigation of the consequences of the use of different fractional derivative definitions:

Example-projects (because they will have a large impact on application level):

1. Determination of the multiplets of the fractional extension of the rotation group SO(n) using the Riesz or regularized Liouville derivative.

2. Investigation of the properties of the fractional extension of the Poincare-group (SUSY and beyond).

3. Compare the influence of different fractional derivative definitions on group structure (e.g. pairing – forces)

4. Is there a connection of the premises of general relativity and fractional calculus (e.g. Lienard-Wiechert-potentials versus Riesz-potentials). Is the (modified) Riesz-potential as a function of (x,y,z,t) Lorenz-invariant? If not, what is the correct form?

1Herrmann, Richard *Fractional Calculus – Introduction for Physicists*, 2nd ed., World Scientific (2014) and references therein