Open problem: simulation of Havriliak–Negami models

Experiments have shown that electric polarization in inhomogeneous materials, such as biological tissues, follows non–standard laws of Havriliak–Negami type

$$Y(s) = H(s) * F(s) \qquad H(s) = \frac{1}{(s^{\alpha} + \lambda)^{\gamma}}$$

Important applications: description of interactions between electromagnetic waves and biological bodies (e.g., treatment and diagnosis of tumors)

In the time–domain Havriliak–Negami models are represented by means of a new pseudo–fractional differential operator

$$\left({}_0D_t^{\alpha} + \lambda\right)^{\gamma} y(t) = f(t)$$

and different definitions have been proposed for this operator, see [Nigmatullin and Ryabov, *Physics Solid State*, 1997] and [Novikov et al., *Mater. Sci. Poland*, 2005]

A challenging computational problem: device and study convolution quadratures, for the numerical discretization of the differential operator $\left({}_0D_t^{\alpha} + \lambda\right)^{\gamma}$, in the form

$$y_n = \sum_{j=0}^{n} \omega_{n-j}(\alpha, \gamma, \lambda) f_j$$